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Collective Defined Contribution Plans – Backtesting Based on German Capital Market Data 1950 - 2022

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Abstract

Using historical capital market data for Germany (1950-2022) we analyze and compare (*individual*) *defined contribution* (*IDC*-) and *collective defined contribution* (*CDC*) pension plans. To this end we define simple asset liability management rules that govern a *CDC* pension plan and compare these to *IDC*-plans with the same asset allovation. Our main result is, that the *CDC* pension plans allow for a significant improvement of the risk return profile compared to individual pension plans. Hereby we consider different risk measures. This empirical study affirms the theoretical results based on stochastic *CDC*-models.

1. Introduction

Pension systems worldwide consist of a combination of *Pay-as-You-Go* (*PAYG*) *systems* and *capital-funded* systems. From the macroeconomic viewpoint a *PAYG* system is linked to the *production factor labor* while a capital-funded systems is linked to the *production factor capital*. Both systems have their merits and drawbacks. One argument in favor of capital funding is the fact that in most developed countries we observe an aging working population, that puts pressure on *PAYG*-Systems. However, a pension fund cannot directly invest into the production factor capital in the notion of national income accounting.[1] In particular an investment into corporate bonds only indirectly secures a share in capital stock of an economy since part of the profit generated by the company, namely a risk premium, goes to the shareholders. Similar considerations apply to government bonds. Thus - at least in theory - only a 100% real investment (stocks and real estate) would insure a full participation in the production factor capital. But a 100% equity investment means an unacceptable risk for most savers. So, capital funded pension systems are confronted with the *risk-return dilemma*: The capital can be invested in risky asset (such as equities) with high returns or can be invested safely (e.g. in government bonds) with significant lower returns. The *equity risk premium* (*ERP*) is a well-established key figure to gauge the *average* extra return if one invests in equities rather than in bonds. However, an individual saver who puts aside part of her/ his labor income during working life may face a stock market crash just at the moment she/ he wants to retire or to buy an annuity. The problem is that an individual saver (the same applies to an age cohort of savers) cannot realize the *average* return on equities.

Collective defined contribution (*CDC*) pension systems try to overcome this problem by some kind of collective agreement among generations of savers aiming to redistribute “deviations” from the average. [2] The idea of intergenerational risk transfer with respect to capital market risks is old since classical with-profit life insurance or endowment policies can be regarded as an implementation of some kind of intergenerational (capital market) risk transfer.[3] The last years research into intergenerational risk transfer arrangements has helped to better understand *CDC* pension schemes. Applying stochastic models (including stochastic simulation techniques) one can prove a positive welfare effect (e.g. Gordon/ Varian 1988,

[1] Only a small fraction of the national capital (machinery, real estate, patents, intellectual property, …) are traded on capital (including real estate) markets.
[2] Cf. Gollier (2008)
[3] Cf. Goecke (2003)

Gullién/ Jørgensen/ Nielsen 2006, Gollier 2008, Hoevenaars 2008 etc.) of *CDC*-arrangement or we can derive optimal ALM-strategies (Chen/ Kanagawa/ Zhang (2021)). However, to the best knowledge of the author, there are only few empirical studies with respect to *CDC* plans (Wesbroom/ Arends/ Turnock/ Harding 2015 with UK-Data). We want to fill this "empirical gap" by a backtesting based on German capital market data. Even though we are working with real world capital market data it should be clear, that the following is not a study of real *CDC*-pension systems; it is rather a study about how a *CDC*-pension system would have performed if as proper *CDC*-system had been installed in the past decades.

The author is well aware of the fact that the following study only refers to the German market and that historic market data may not be representative for what might happen in future. However, one should keep in mind that even the most elaborated stochastic model is equally limited with respect to the predictive power – the future is only a realization of one of millions of possible future random walks![4]

The *CDC*-model underlying our backtesting is inspired by the continuous time (*c.t.*) model presented in Goecke (2013) and we want to check whether the theoretical results derived for the *c.t.* model remain valid for real capital markets. To this end firstly, we have to formalize a discrete time (*d.t.*) version and secondly, we have to select proper capital market data.

To differentiate from *CDC*-models we will refer to (*individual*) *defined contribution* (*IDC*-) models, where the saver participates in a normal investment fund arrangement, where the individual pension capital is one-to-one linked to the market value of assets of that particular investment fund.

[4] We do not discuss the rather philosophical question whether stochastic models build on the mathematical concept of probability are in principle appropriate to model economic scenarios; the fundamental problem is that the mathematical concept of probability implicitly presumes that uncertain events (like throwing dices) can be repeated arbitrarily often and that economic scenarios are not the result of throwing dices but mainly result of human decisions, partly driven by emotions and irrationalities.

Capital Market Data for Backtesting

The *c.t.* model in Goecke (2013) presumes a stylized capital market with a risk-free asset with a constant interest rate and a risky asset driven by a geometric Brownian motion with constant drift and volatility. It is assumed that the pension assets are a mix of the risk-free and the risky asset and that the *equity ratio* (i.e. the relative part of risky assets) can be adjusted continuously. If we want to check the theoretical time continuous model with real capital market data, we must find a suitable proxy for a risk-free asset and a risky asset. For our back-testing we assume that for a pension manager an investment into top rated government bonds is regarded as a risk-free investment and that an investment into a well-diversified portfolio of equities is regarded as a risky investment with a positive extra return. We take the German government bond index *REXP* as a proxy of a risk-free bond portfolio and the German equity index *DAX* a proxy of a broadly diversified equity portfolio.

Clearly, an investment into a *REXP*-Portfolio is not risk-free in the strict sense since from month to month we observe unexpected gains or losses from volatile market interest rates leveraged by the duration of the portfolio. Consistent with our pragmatic definition of a risk-free asset, we will substitute the constant risk-free interest rate of the time continuous model by the current yield of outstanding government bonds $i_s(t)$, which we interpret as the expected return of a bond investment. The time continuous model implicitly presumes a zero-inflation economy. We therefore adjust our real capital market data to eliminate inflation. To this end we take a suitable consumer price index $CPI(t)$ and calculate price adjusted index values $REXP_p$ and DAX_p. On the basis $i_s(t)$ we define $\mu_s(t)$, the price adjusted *expected* (log-)*return* (based on information up to time t) for the month following t of a risk-free investment. Details are explained in **Appendix**.

2. A simple *CDC* model for backtesting

2.1. Model description

The *CDC* model for our backtesting is a *discrete time* (*d.t.*) adoption of the *continuous time* (*c.t.*) model presented in Goecke (2013). The discrete time interval is $\Delta = {}^1/_{12}$ i.e. one month. We consider a pension fund with assets and liabilities. The assets at time t – denoted $P(t)$ – are invested in a mixture of equities (represented by DAX_p) and bonds (represented by $REXP_p$). The liability side of the balance sheet of our pension fund consists of two parts: the total of all individual accounts – denoted by $V(t)$ – and a capital reserve – denoted by $R(t)$ – which serves as a buffer. We have $P(t) = R(t) + V(t)$; we assume that always $P(t) > 0$ and $V(t) > 0$, but we allow for a negative reserve $R(t) < 0$ in case of an underfunding of the pension fund.

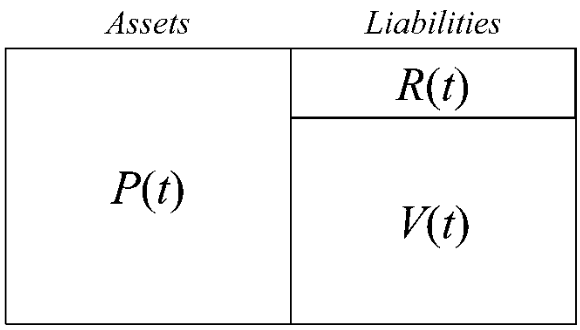

Figure 1. Balance sheet of a *CDC* pension funds

We assume that at the beginning of each month a new generation enters working life and starts paying contributions into the pension fund. At the same time the generation of freshly retired leaves the pension fund taking with them their individual assets, i.e. their share of $V(t)$. We thus do not consider lifetime annuity payments to retirees, instead we assume that all retirees receive a one-off payment upon retirement. We implicitly assume that all savers live to see their retirement.

We assume that the sum of contribution of all active workers just matches the total of money paid out to the retirees. By this assumption we rule out dynamic effects from a growing or shrinking population. We do not rebuild the start-up phase of a *CDC* system; instead we start with an initial balance sheet $\left(P(t_0), V(t_0), R(t_0)\right)$ in $t_0 = 0$ representing01.01.1950. However, what we do is we simulate more or less favorable starting conditions by setting the initial reserve ratio at different levels. As pointed out, we use *price adjusted data*. Thus, 1 Euro contribution payed by a generation of active workers has the same real value as 1 Euro paid out to the same generation decades later.

Notation and Definitions

Our *d.t.* model is based on monthly data, i.e. our time unit Δ represents one month. The time index t represents the first day of the months within the backtesting period (01.01.1950 to 01.07.2022). Generally, we identify the *end* of a month with the *beginning* the following month. For example, if we consider a 480-month saving plan starting at the *beginning* of Jan. 1960 and falling due at the *end* of December 1999, we say that the saving plan starts at $t = 120\Delta$ and ends at $t = 600\Delta$.

At time t, the beginning of month $[t, t{+}\Delta[$, the pension manager determines the equity ratio $\beta(t) \geq 0$. Within the time interval $[t, t+\Delta[$ we follow a *buy-and-hold strategy*, i.e. with respect to the performance of $P(t)$ we assume that

$$\frac{P(t+\Delta)}{P(t)} = \beta(t)\,\frac{DAX_p(t+\Delta)}{DAX_p(t)} + \left(1-\beta(t)\right)\frac{REXP_p(t+\Delta)}{REXP_p(t)} \qquad \text{for } t \geq 0. \qquad \textbf{(Eq 1)}$$

For $t \geq \Delta$ we define $\mu_P(t) := \ln\left(\dfrac{P(t)}{P(t-\Delta)}\right)$.

Furthermore, at time $t \geq 0$ the pension manager determines the *declaration* $\eta(t)$, namely the (log-) interest rate for the month $[t, t+\Delta[$, by which the individual accounts are updated, i.e.

$$V(t+\Delta) = V(t)\exp\big(\eta(t)\big) \text{ or } \eta(t) = \ln\left(\frac{V(t+\Delta)}{V(t)}\right).$$

Note that $\mu_P(t)$ can be observed at time t and refers to the investment period $[t-\Delta, t\,[$, while $\eta(t)$ refers to $[t, t+\Delta[$. $\beta(t)$ and $\eta(t)$ are determined at time t based on the information available up to time t . Thus $\beta(t)$, $\eta(t)$ and $\mu_P(t)$ can be regarded as t-adapted processes.

We define $\rho(t) := \ln\left(\frac{P(t)}{V(t)}\right) = -\ln\left(1 - \frac{R(t)}{P(t)}\right)$ and call it *reserve ratio* at time t. We prefer this log-definition instead of $R(t)/P(t)$ because it simplifies the notation. For example, using this definition we get the following simple recursion for the reserve ratio:

$$\rho(t+\Delta) = \rho(t) + \mu_P(t+\Delta) - \eta(t). \qquad \textbf{(Eq 2)}$$

The fact that $\eta(t)$ is determined at time t while $\mu_P(t+\Delta)$ cannot be observed before $t+\Delta$ implies that the *ALM*-manager cannot guarantee a positive reserve ratio.

The *ALM*-strategy proposed in Goecke (2013) and analyzed in Chen e.a. (2021) is characterized by the following three rules:

Rule 1: The pension manager tries to keep the reserve ratio $\rho(t)$ close to a given *strategic reserve ratio* $\rho_S \geq 0$; we call $\hat{\rho}(t) := \rho(t) - \rho_s$ the *reserve gap*.

Rule 2: The pension manager follows a given *strategic equity ratio* $\beta_S \in [0, 1]$. However, we allow for monthly adjustments of $\beta(t)$ as a reaction of a positive or negative reserve gap.

Rule 3: The declaration $\eta(t)$ should basically follow the *expected rate of return* of the pension assets, in particular a high equity ratio should result in a higher declaration. However, because of *Rule 1*, the actual declaration is adjusted according to the reserve gap.

These general rules must be specified. Again, we follow the approach in Goecke (2013) and define an *asset management rule* (*AM*-rule) and *liability management rule* (*LM-rule*) depending on parameters $\alpha_\Delta \geq 0$ and $\theta_\Delta \geq 0$:

AM-Rule: $\beta(t) = \beta_s + \alpha_\Delta \left(\rho(t) - \rho_s\right) = \beta_s + \alpha_\Delta \, \hat{\rho}(t)$ with side constraint $0 \leq \beta(t) \leq 1$, [5]

LM-Rule: $\eta(t) = \mu_P^e(t) + \theta_\Delta \left(\rho(t) - \rho_s\right) = \mu_P^e(t) + \theta_\Delta \, \hat{\rho}(t)$,

where $\mu_P^e(t)$ denotes the *expected rate of return* of the assets based on the information up to time t. We define $\mu_P^e(t) := \mu_s(t) + \Delta\left(\beta(t)\, ERP - \frac{1}{2}\beta^2(t)\,\sigma_M^2\right)$, where $\mu_s(t)$ denotes the expected return of a risk-free investment, *ERP* is the *equity risk premium* and σ_M is an estimate of the *average market volatility*. We take *ERP* = 5% and σ_M = 20% - see Appendix 1 for details. We should keep in mind that the calibration of *ERP* and σ_M only has effect on the *expected return* on assets; the actual performance of the pension portfolio $P(t)$ is not affected by the choice of $\eta(t)$. However, if we overestimate *ERP*, then the declaration $\eta(t)$ tends to be too optimistic and the probability on a negative reserve gap will increase.

Given *ERP* and σ_M the asset liability management is fully determined by only four parameters, namely β_s, ρ_s, α_Δ and θ_Δ.

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[5] We could allow for $\beta(t) > 100\%$, which corresponds to a debt-financed investment in equities. However, in the context of pension management a debt- financed speculative investment makes little sense apart from the fact that this kind of investment might be prohibited by supervisory authorities.

Inserting $\eta(t) = \mu_P^e(t) + \theta_\Delta \, \hat{\rho}(t)$ into **(Eq 2)** we get

$$\hat{\rho}(t+\Delta) = (1-\theta_\Delta)\hat{\rho}(t) + \mu_P(t+\Delta) - \mu_P^e(t). \qquad \textbf{(Eq 3)}$$

If $0 < \theta_\Delta < 1$ and provided that $\mu_P^e(t) = \mathbb{E}\left(\mu_P(t+\Delta) \middle| \mu_P(t)\right)$ then *on average* the reserve gap $\hat{\rho}(t)$ is reduced by factor (1-θ_Δ), i.e. the (discrete) stochastic process $\rho(t)$ is *mean-reverting* at level ρ_s and θ_Δ is the speed factor.[6]

We can interpret θ_Δ as the *intergenerational risk sharing parameter* since θ_Δ controls to what degree a mismatch between the strategic reserve ratio and the actually observed reserve ratio is transferred to the next generation. If e.g. $\theta_\Delta = 0$ we have $\hat{\rho}(t+\Delta) = \hat{\rho}(t) + \mu_P(t+\Delta) - \mu_P^e(t)$, which means that any systematic misestimation of $\mu_P(t+\Delta)$ is carried forward *ad infinitum.* On the other hand, if $\theta_\Delta = 1$ then $\hat{\rho}(t+\Delta) = \mu_P(t+\Delta) - \mu_P^e(t)$, i.e. the misestimation $\mu_P(t+\Delta) - \mu_P^e(t)$ is carried forward only one period. This means that any asset shock is fully compensated by a corresponding adjustment of the declaration at the beginning of the next period; i.e. $\eta(t+\Delta) = \mu_P^e(t+\Delta) + \mu_P(t+\Delta) - \mu_P^e(t) = \mu_P^e(t+\Delta) + \hat{\rho}(t+\Delta)$.

2.2. Data

2.2.1. Capital Market Data and Calibration[7]

Backtesting period: 01.01.1950 ($t = 0$) – 01.07.2022 ($t = 870\Delta$)

Input data: $DAX_p(t)$, $REXP_p\,(t)$ and $\mu_s(t)$ für $t = 0, \Delta, 2\Delta, \ldots, 870\Delta$

$ERP = 5.0\%$; $\sigma_M = 20\%$

[6] Cf. Goecke [2013] Proposition A.2.

[7] See Appendix for details

Parameters: ρ_s: strategic (log-) reserve ratio

β_s: strategic equity share (risk exposure)

θ_Δ : adjustment (speed) parameter for profit participation

α_Δ : adjustment (speed) parameter for asset allocation

ρ_0: initial (log-) reserve ratio, $\rho(0) = \rho_0$

ALM-rules: $\beta(t) = \beta_s + \alpha_\Delta \hat{\rho}(t)$, subject to $0 \leq \beta(t) \leq 1$

$\eta(t) = \mu_P^{(e)}(t) + \theta_\Delta \hat{\rho}(t)$,

with $\mu_P^e(t) = \mu_s(t) + \Delta\left(\beta(t) ERP - \frac{1}{2}\beta^2(t)\sigma_M^2\right)$.

In particular, for β_s =100% and α_Δ =0 we have $\mu_P^e(t) = \mu_s(t) + 0.03\,\Delta$. Note that the *ALM*-rules only depend on the reserve gap $\hat{\rho}(t) = \rho(t) - \rho_s$, so that the *CDC*-variants are determined by only four parameters, namely $\left(\beta_s, \theta_\Delta, \alpha_\Delta, \hat{\rho}(0)\right)$.

2.2.2. Risk measures [8]

Our aim is to compare *CDC*- and *IDC*-plans with a focus on the risk return profile on long term saving plans. We restrict ourselves to 40-year[9] savings plan with a constant saving rate of 1 money unit payable at the beginning of each month. Since we use price adjusted capital market data, we implicitly assume the saving rates have always the same purchasing power. Before we can proceed analyzing long term saving plans, we define some key indicators to measure return and (certain aspects) of risk. We do not enter into a discussion how to measure the risk of long-term saving plan, we just present different (statistical) risk measures which

[8] A comprising survey of risk measures can be found in Bacon (2022).
[9] 40 years represent the duration of a working life, say from age 25 to age 65.

are plausible and mirror at least aspects of risk. One should keep in mind that risk is a very complex phenomenon and all statistical methods have only limited explanatory power.

Given an investment vehicle (e.g. an *IDC*-plan or a *CDC*-plan) let us denote ν_t the log-return for the month $[t-\Delta, t]$ for $t = \Delta, 2\Delta, \ldots, 870\,\Delta$. For a fixed n (e.g. $n = 480$) we denote by SP_t the saving plan starting at time t and maturing in $t + n\,\Delta$ for $t = 0, \Delta, 2\Delta, \ldots, (870-n)\,\Delta$. By $SP = \{SP_0, SP_\Delta, SP_{2\Delta}, \ldots, SP_{(N-n)\Delta}\}$ we denote the set of (overlapping) saving plans within our back-testing period $[0, 870\,\Delta]$ - for $n = 480$ we have 391 saving plans.

Denote by $SP_t(k)$ the accrued capital of saving plan SP_t after k ($0 \le k \le n$) months; i.e. $SP_t(0) = 0$ and $SP_t(k) = \left(SP_t(k-1)+1\right)\exp(\nu_{t+k})$ for $k = 1,\ldots, n$. The *rate of return at maturity* of SP_t is the interest rate $r = r\,(SP_t)$ which solves $SP_t(n) = \sum_{k=1}^{n}(1+r)^{k\Delta}$.

The ***average rate of return*** of SP is $r(SP) := Mean\left\{r(SP_0), r(SP_\Delta), \ldots, r(SP_{(N-n)\Delta})\right\}$

The ***average intergenerational imbalance*** (***IGI***) of SP is defined by

$$IGI_{mean}(SP) := mean\left(1 - \frac{SP_t(n)}{\max\left\{SP_t(n), SP_{t-\Delta}(n), \ldots, SP_{t-12\Delta}(n)\right\}} : t = 12\Delta,\, 13\Delta,\, \ldots, (N-n)\Delta\right)$$

The ***maximum IGI*** defined by

$$IGI_{max}(SP) := max\left(1 - \frac{SP_t(n)}{max\left\{SP_t(n), SP_{t-\Delta}(n), \ldots, SP_{t-12\Delta}(n)\right\}} : t = 12\Delta,\, 13\Delta,\, \ldots, (N-n)\Delta\right)$$

The idea behind this risk indicator is that a person who retires (i.e. whose saving plan matures) at time t compares her or his pension capital with the pension capital of those who retired within the last year. If this person has a pension capital of say 20,000 and 7 months ago the retires got 25,000, then this person has 20% less and feels unfairly treated by "the system". Keeping in mind that from the macroeconomic point of view a capital funded pensions system should provide a fair participation in *capital* as a *factor of production*, it is obvious that with 7 months the value of this participation should not drop by as much as 20%.[10] Note that *IGI*(*SP*) solely depends on the final capital.

In the following we define path dependent risk indicators – i.e. we evaluate the whole accumulation period and not only the final capital.

The ***volatility*** of SP_t is defined as the annualized standard deviation of the log-return during the saving period: $Vola(SP_t) := \sqrt{12}\ StDev\left(\{v_{t+\Delta}, v_{t+2\Delta}, ..., v_{t+n\Delta}\}\right)$. A low value for $Vola(SP_t)$ indicates a stable building up of the pension capital. The ***average volatility*** over all pension plans within the backtesting period is then

$$Vola(SP) := Mean\left\{Vola(SP_0), Vola(SP_\Delta), ..., Vola(SP_{(N-n)\Delta})\right\}.$$

From the saver's point of view the volatility has only a limited information value. Firstly, volatility is an abstract figure, which only can be understood with some mathematical background. Secondly, for saving plans the impact of high or low volatility is very much depending on when the volatility occurs: Volatile returns during the first years of saving can be compensated by an adjustment of the saving rates, while towards the maturity date this is nearly impossible.

The ***maximum drawdown (MDD)*** [11]of SP_t is defined as maximal relative loss during the saving period: $MDD(Sp_t) := Max\left\{1 - \frac{Sp_t(l)}{Sp_t(k)} : 1 \le k \le l \le n\right\}$. $MDD(SP_t) = 0$ indicates that at any time the accumulated capital never drops below a precedented value. Since we are

[10] The problem is that the market value of an asset is always reflecting the *estimated* future returns from this asset and that these estimations are not only driven by fact but also by sentiments and "*irrational exuberances*" – c.f. Shiller (2014).
[11] Mahmoud (2015)

evaluating saving plans with constant saving rates, typically the drawdown will be zero at the beginning of the saving process and will be maximal towards the end of the saving process.

The ***average MDD*** of *SP* is defined

$$MMD(SP) := Mean\left\{MDD(SP_0), MDD(SP_\Delta), ..., MMD(SP_{(N-n)\Delta})\right\}.$$

The ***maximum loss duration (MLD)*** of SP_t is defined as maximal number of months within the saving period in which the pension capital does not increase - despite regular contributions. Formally defined as $MLD(SP_t) := \max\left\{l-k : 1 \leq k < l \leq n \;\; and \;\; SP_t(k) < SP_t(l)\right\}$.

The ***average MLD*** over all pension plans is defined as

$$MLD(SP) := Mean\left\{MLD(SP_0), MLD(SP_\Delta), ..., MLD(SP_{(N-n)\Delta})\right\}.$$

We illustrate our risk indicators for *IDC*-saving plans for a constant mix portfolio of equities (represented by DAX_p) and secure bonds (represented by $REXP_p$) with equity ratio $\beta = 0\%$, 50% and 100%. There are in total 391 overlapping saving plans - the first ending 31.12.1989 ($t = 480\Delta$), the last ending 30.06.2022 ($t = 870\Delta$).

To evaluate the risk of a certain investment strategy (e.g. by choosing the equity ratio) we could restrict ourselves looking at the final capital or, equivalently, the rate of return at maturity. A good argument for this view is that only the final capital matters since it determines the affordable annuity. In view of Figure 2 we would then conclude that a 100% equity ratio is the best saving strategy, since only at two maturity dates (28.02.2003 and 31.03.2003) the return of a 100% equity investment was a bit worse than a “secure” investment into government bonds.

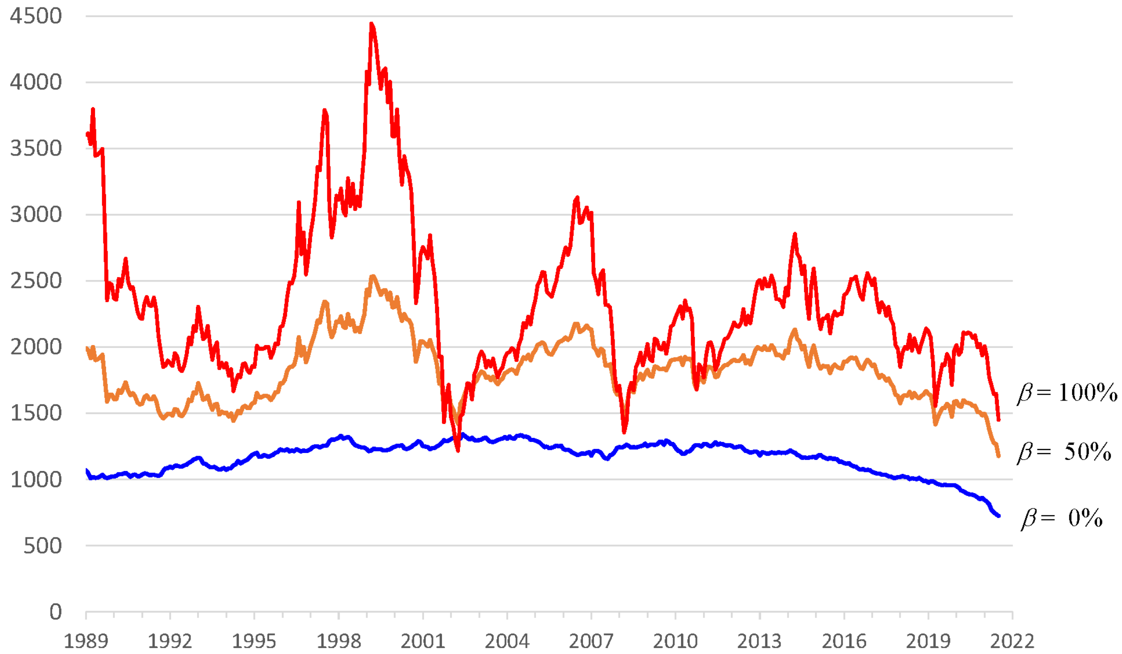


Figure 2. *Final capital* of 480-months *IDC*-saving plans maturing between 31.12.1989 and 30.06.2022 for $\beta = 0\%$, 50% and 100%

Figure 2 also illustrates that a high equity ratio is risky in the sense that the final capital is very "volatile". A risk indicator for the "volatility" would be the *standard deviation* of the final capitals (or the rates of return) for the different investment vehicles. However, we think that in this case the *standard deviation* is not a proper risk measure. To see this, we have to note that the standard deviation of a time series measures the deviation from the *average* as the point of reference. Recalling that a capital funded system should ideally ensure a fair participation in the productivity of invested capital, it is obvious that over a long period (in our case 72 $^1/_2$ years!) the productivity changes driven by technology, skills, political stability, natural resources, etc. So, a perfect capital funded system would not ensure a constant final capital, but a final capital that reflects the economic progress.

That is the reason why we prefer the *intergenerational imbalance* as risk indicator. We illustrate this risk indicator by Figure 2: For a $\beta = 100\%$ equity portfolio we calculate $ICI_{max} = 57.24\%$, since the final capital for the saving plan maturing on 31.03.2003 is

1216.64, which is 57.24% lower that the final capital of the saving plan maturing 12 months before (2845.21). The average over all maturing dates is ICI_{mean} = 11.52%. For a β = 50% equity portfolio we get ICI_{max} = 30.86% and ICI_{mean} = 6.06%, and for β = 0% we get ICI_{max} = 18.24% and ICI_{mean} = 2.95%. We see that even for a presumed secure investment in German government bonds within only 12 months the pension capital of two generations of savers can differ by more than 18%. The reason for this is, that between June 2021 and June 2022 the interest rates *and* inflation rates increased substantially.

As pointed out, we believe that to assess the risk of a certain pension vehicle (and hence the acceptance by consumers) it is not enough to evaluate the *final* capital but also the *whole accumulation period*. The ups and downs during the accumulation period definitely can stress the saver – in other words we think that a saving plan with heavy ups and downs are perceived to be riskier than a saving plan with a continuously increasing accrued capital.

To illustrate this, we look at the 40-years saving plan that starts on 01.04.1963 and ends on 31.03.2003 – cf. Figure 3.

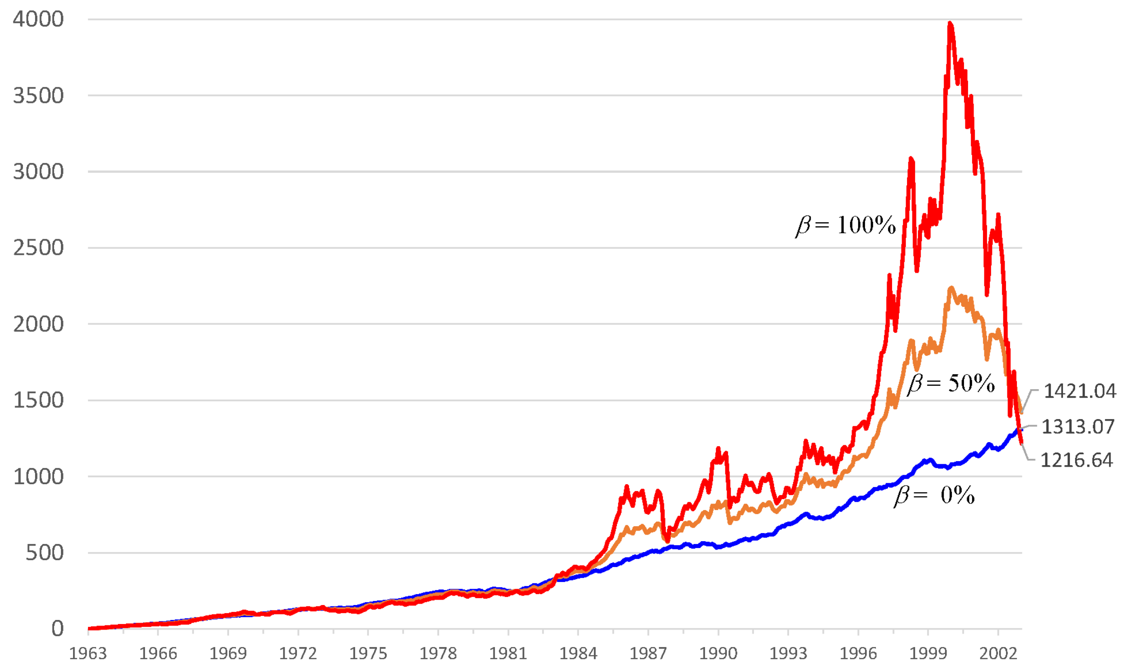


Figure 3. Accrued capital of a saving plan (1.4.1963-31.3.2003) for β = 0%, 50%, 100%.

We see that the final pension capital is more or less the same for all three levels of the equity ratio.[12] However, investing into $REXP_p$ ($\beta = 0$) results in a more or less continuously increasing accrued capital, while a DAX_p ($\beta = 100\%$) investment makes any reasonable projection of the final pension capital impossible. The *volatility* of the monthly returns[13] during the investment period is 3.87% ($\beta = 0\%$), 9.23% ($\beta = 50\%$) and 17.77% ($\beta = 100\%$).

The risk indicators *MDD* and *MLD* are illustrated in Figure 4. Figure 4 depicts the accumulation period for saving plans maturing end of June 2022 for $\beta = 0\%$ and $\beta = 100\%$. The *MDD* of the $\beta = 100\%$ saving plan is 67.92%, since between end of Febr. 2000 and end of March 2003 the accrued capital fell from 931.87 to 298.95 - by 67.92%. The *MDD* for $\beta =$ 0% is 16.71%, respectively.

The *MLD* for $\beta = 100\%$ and 0% is (respectively) 202 and 121 months. The *MLD* for $\beta = 0\%$ is illustrated in Figure 4: the accrued capital end of May 2012 is 728.05 and 121 months later (end of June 2022) it is still lower, namely 725.14. The extreme high loss duration for a "safe" investment vehicle is caused by low (price adjusted) interest rates during most of the saving period, the increasing interest rates combined with high and inflation in 2022. Still true is that a risk-averse person paid 121 contributions while at the same time the purchasing power of her of his pension capital did not grow.

[12] The rate of return is 4.48% (for $\beta = 0\%$), 4.81% (for $\beta = 50\%$) and 4.18% (for $\beta =100\%$)
[13] Annualized standard deviation of 480 monthly (log-) returns

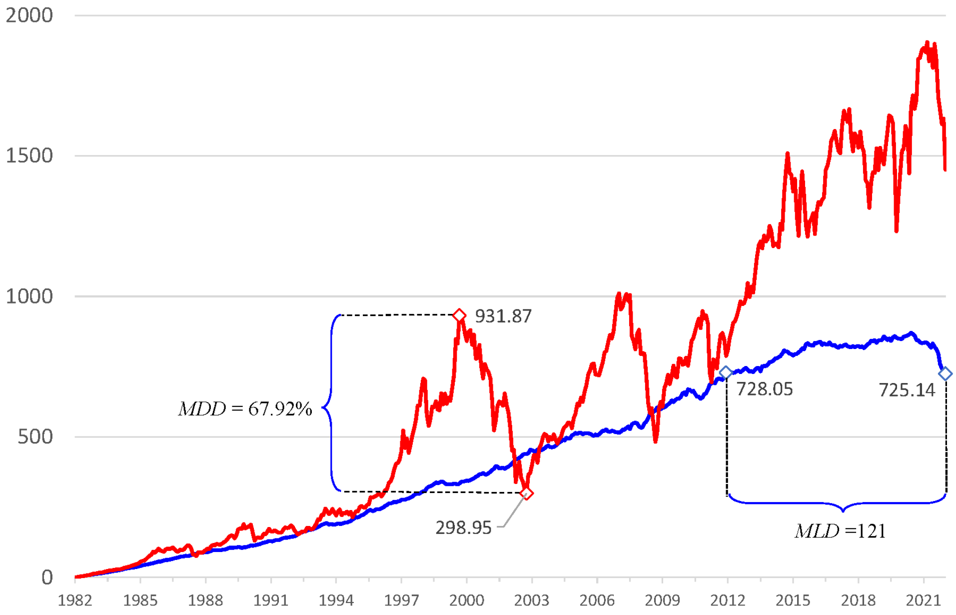


Figure 4. Accumulation period of a saving plan (1.7.1982-30.6.2022) for $\beta = 0\%$ and 100%.

3. Results

3.1.1. Lump sum investment

Before evaluating the saving plans we illustrate the effect of the intergenerational risk transformation of *CDC*-plans. To this end evaluate a lump-sum investment for the total back-testing period (01.01.1950 to 30.06.2022) for different investment vehicles:

- *Constant mix portfolio* with a share of β invested into DAX_p and (1- β) invested into $REXP_p$ portfolio for $\beta \in \{0\%, 10\%, \ldots, 100\%\}$ – abbreviated *CM*(β).
- Investing into a *CDC*-Portfolio with $\theta_\Delta = 2\%$, $\alpha_\Delta = 0$, $\hat{\rho}(0) = 0$ and with an underlying constant mix portfolio with $\beta \in \{0\%, 10\%, \ldots, 100\%\}$ - abbreviated *CDC*(θ_Δ= 2%, β)
- Investing into a *CDC*-Portfolio with $\theta_\Delta \in \{0.5\%, 1\%, 2\%, 5\%, 10\%, 20\%, 40\%, 60\%$ $80\%, 100\%\}$, $\alpha_\Delta = 0$, $\hat{\rho}(0) = 0$ with $\beta = 100\%$ - abbreviated *CDC*(β=100%, θ_Δ).

For each portfolio we evaluate the rate of return and the annualized volatility. The annualized volatility (***Vola***) is calculated as $\sqrt{12}\cdot StDev\left(\ln\left(\frac{P(t+\Delta)}{P(t)}\right): 0 \le t \le T-\Delta\right)$ for constant mix portfolio investment and as $\sqrt{12}\cdot StDev\left(\ln\left(\frac{V(t+\Delta)}{V(t)}\right): 0 \le t \le T-\Delta\right)$ for investment into $V(t)$.

To start with, Figure 5 just illustrates the interplay of $\mu_P(t)$, $\eta(t)$, and the reserve gap $\hat{\rho}(t)=\rho(t)-\rho_s$. In this example the underlying portfolio is a DAX_p investment (β=100%) and the parameters for the *CDC*-model are $\left(\beta_s, \theta_\Delta, \alpha_\Delta, \hat{\rho}(0)\right) = (100\%, 2\%, 0, 0)$.

The smoothing effect of the intergenerational risk transfer is obvious; the annualized volatility of the $\mu_P(t)$-paths is 17.83% and 2.05% for the $\eta(t)$-paths – a reduction of more than 88%! This return smoothing is only possible if we allow for a volatile reserve gap. Since for β_s = 100% and α_Δ =0 $\eta(t) = \mu_P^e(t) + \theta_\Delta\,\hat{\rho}(t) = \mu_s(t) + 0.03\Delta + \theta_\Delta\,\hat{\rho}(t)$ we get as a rough estimation $StdDev\left(\eta(t)\right) \approx \theta_\Delta\; StdDev\left(\hat{\rho}(t)\right)$, if we neglect the impact of the $\mu_s(t)$-term.

Actually, in this example we have $StdDev\left(\eta(t)\right)$ = 0.59% and $\theta_\Delta\; StdDev\left(\hat{\rho}(t)\right)$ = 0.545%.

The mean reverting character of the *ALM*-rules insure that the reserve ratio always returns to the strategic level. This effect is even more pronounced in practice since equity bear markets have always be followed by bull markets and vice versa.

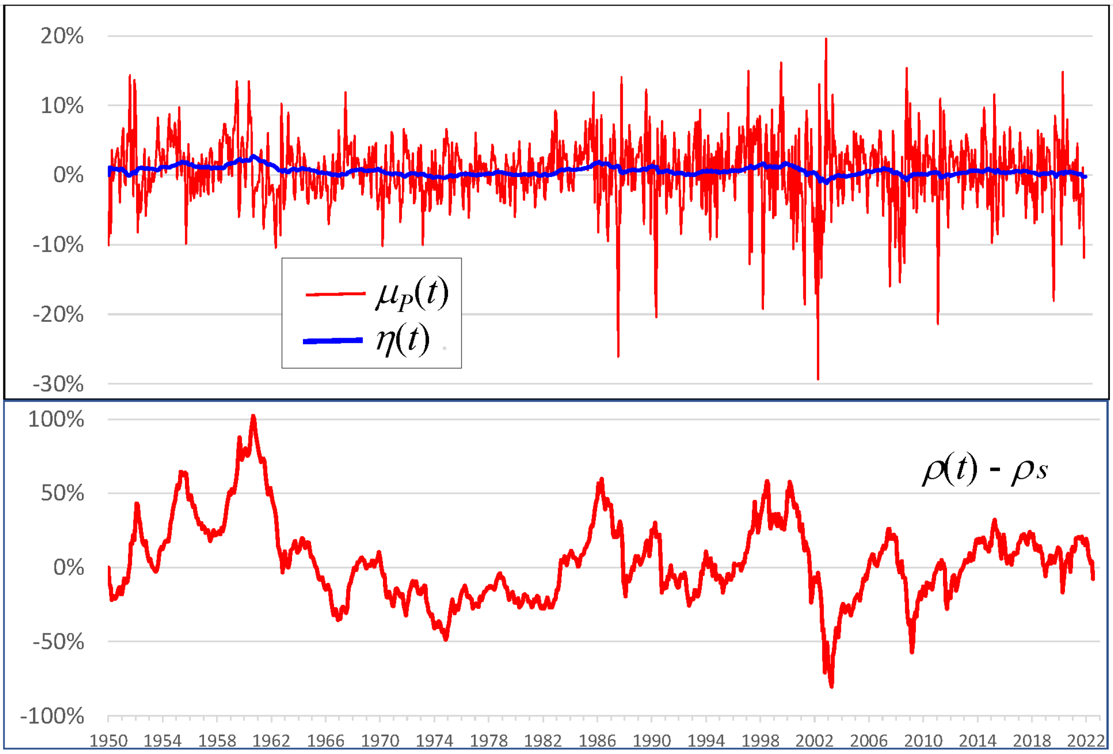


Figure 5. $(\mu_P(t), \eta(t), \rho(t) - \rho_s)$ for $0 \leq t \leq T$

For the three types of investment vehicles explained above ($CM(\beta)$, $CDC(\theta_\Delta = 2\%, \beta)$, $CDC(\beta = 100\%, \theta_\Delta)$) we calculate risk-return profiles, where risk is measured as the volatility and the return as the (annualized) rate of return over the back-testing period. The volatility-return profile for $CM(\beta)$ confirms the *risk-return dilemma*, i.e. that an increasing (expected) return comes along with increasing risk (here: volatility).

As pointed out the investment in a pure $REXP_p$ portfolio is not risk free with respect to volatility. However, volatility – as defined here - might overstate the investment risk, if we want to evaluate long-term investments. DAX_p and $REXP_p$ are not fully correlated, so there is a diversification effect between both.

The risk-return profile for $CDC(\theta_\Delta= 2\%, \beta)$ shows the risk-mitigating effect of buffering capital market variations. There is also a risk-mitigating effect for a pure $REXP_p$ -portfolio (β = 0) since the collective reserve buffers short term interest rate fluctuations.

The rates of return for *CDC*(θ_Δ= 2%, β) are slightly higher than the rates of return for *CM*(β) due to the fact that the final reserve gaps are slightly negative.

The risk-return profile for *CDC*(β=100%, θ_Δ) shows that θ_Δ is the crucial *ALM*-parameter to manage the smoothing effect. We do not include the case θ_Δ = 0 because in this case the reserve gap is just a “random” walk and not mean reverting.

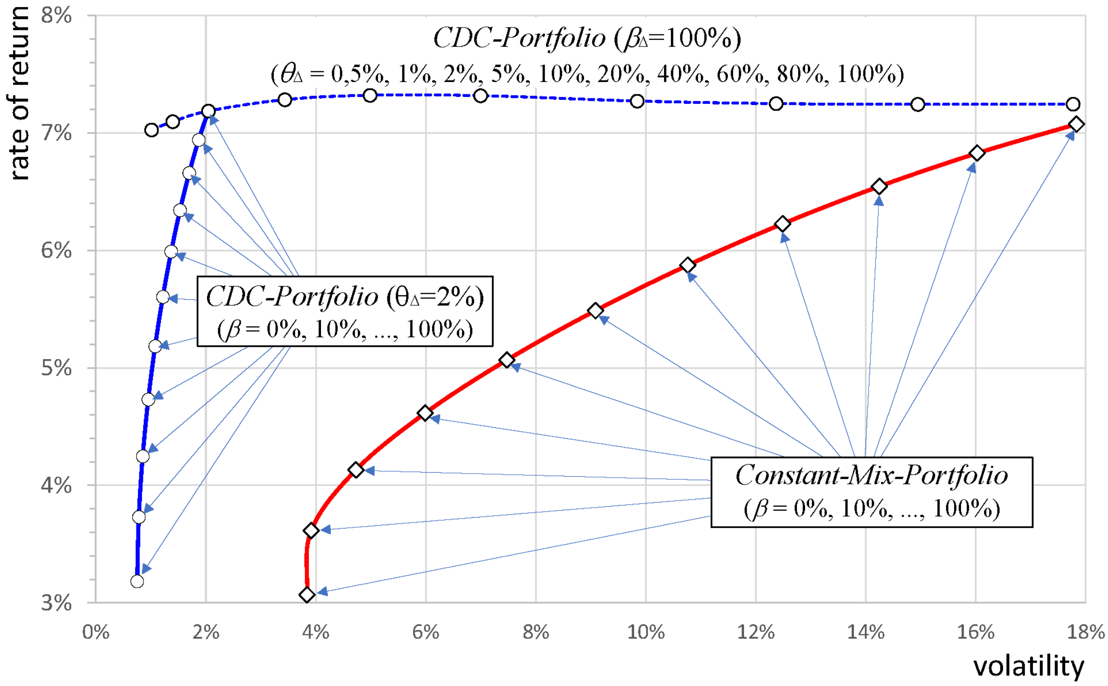


Figure 6. *Return-volatility-profile* of different *IDC-*/ *CDC*-investment vehicles

Const-Mix-Portfolio			*CDC*(θ_Δ = 2%, β)			*CDC*(β = 100%, θ_Δ)		
β	rate of return	*Vola*	β	rate of return	*Vola*	θ_Δ	rate of return	*Vola*
0%	3.07%	3.84%	0%	3.18%	0.75%	0.5%	7.03%	1.01%
10%	3.61%	3.91%	10%	3.73%	0.78%	1%	7.10%	1.39%
20%	4.13%	4.73%	20%	4.25%	0.85%	2%	7.19%	2.05%
30%	4.61%	5.99%	30%	4.73%	0.95%	5%	7.28%	3.43%
40%	5.07%	7.48%	40%	5.18%	1.08%	10%	7.32%	4.98%
50%	5.49%	9.08%	50%	5.60%	1.22%	20%	7.32%	7.00%
60%	5.87%	10.76%	60%	5.99%	1.37%	40%	7.27%	9.84%
70%	6.23%	12.49%	70%	6.34%	1.53%	60%	7.25%	12.37%
80%	6.54%	14.24%	80%	6.66%	1.70%	80%	7.24%	14.94%
90%	6.83%	16.03%	90%	6.94%	1.87%	100%	7.25%	17.77%
100%	7.07%	17.83%	100%	7.19%	2.05%			

Table 1: Data underlying Figure 6

Figure 7 and Table 2 analyze the behavior of the reserve gap. For $CDC(\theta_\Delta = 2\%, \beta)$ the mean of the reserve gap is close to zero asserting the mean- reversion character of the reserve process. Figure 7 also shows that the distribution of the reserve ratio is roughly symmetric. This is not the case if θ_Δ falls below 2%, because then θ_Δ is to low to force the reserve gap back to zero.

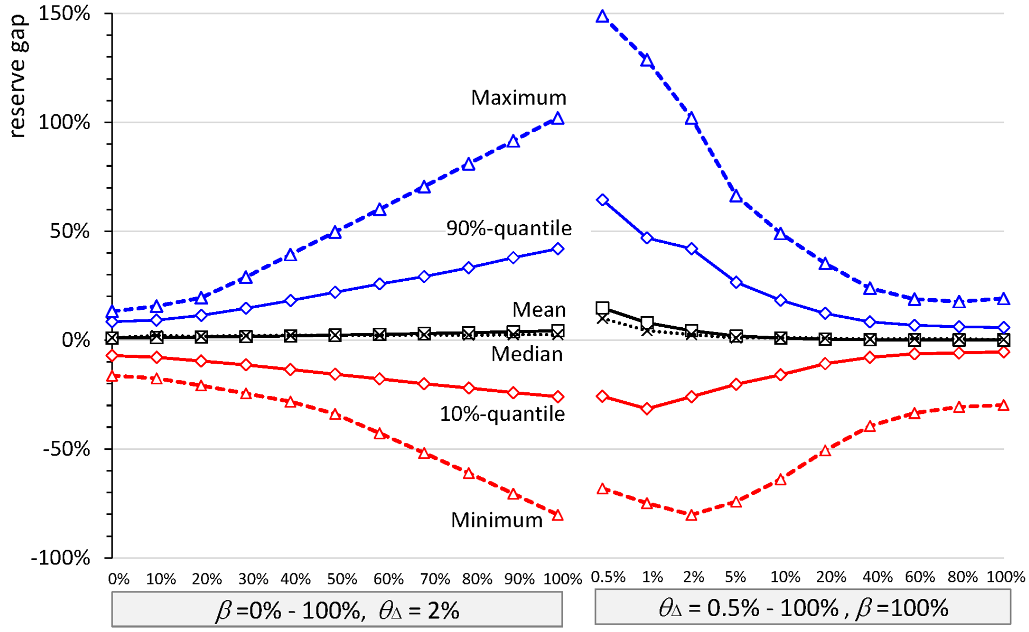


Figure 7. Evaluation of $\{\hat{\rho}(t), t = 0,...,T\}$ for different *CDC*-variants.

	min	max	10%-quantile	90%-quantile	median	mean	StdDev
β	$CDC(\theta_\Delta = 2\%, \beta)$						
0%	-16.30%	13.22%	-7.08%	8.44%	0.96%	1.11%	6.10%
10%	-17.56%	15.57%	-7.89%	9.19%	1.16%	2.06%	6.61%
20%	-20.85%	19.45%	-9.59%	11.38%	1.40%	1.77%	8.00%
30%	-24.51%	28.98%	-11.42%	14.60%	1.67%	2.02%	9.91%
40%	-28.26%	39.34%	-13.47%	18.21%	1.96%	2.13%	12.10%
50%	-33.92%	49.72%	-15.71%	22.00%	2.29%	2.30%	14.45%
60%	-42.75%	60.13%	-17.76%	25.82%	2.64%	2.20%	16.90%
70%	-51.79%	70.57%	-20.06%	29.21%	3.02%	2.44%	19.42%
80%	-61.04%	81.04%	-21.98%	33.21%	3.43%	2.34%	21.99%
90%	-70.52%	91.54%	-24.08%	37.82%	3.86%	2.50%	24.60%
100%	-80.24%	102.08%	-25.97%	41.99%	4.31%	2.57%	27.25%
θ_Δ	$CDC(\beta = 100\%, \theta_\Delta)$						
0.5%	-68.06%	148.90%	-25.83%	64.45%	14.81%	9.96%	37.03%
1%	-74.87%	128.83%	-31.54%	46.95%	7.94%	4.34%	33.14%
2%	-80.24%	102.08%	-25.97%	41.99%	4.31%	2.57%	27.25%
5%	-74.13%	66.37%	-20.31%	26.60%	1.86%	1.04%	19.41%
10%	-63.83%	49.03%	-15.91%	18.37%	0.95%	1.10%	14.33%
20%	-50.69%	35.23%	-10.78%	12.24%	0.46%	0.88%	10.12%
40%	-39.39%	23.85%	-7.89%	8.36%	0.22%	0.43%	7.12%
60%	-33.52%	18.79%	-6.29%	6.83%	0.14%	0.61%	5.97%
80%	-30.65%	17.65%	-5.86%	6.13%	0.10%	0.51%	5.41%
100%	-29.84%	19.15%	-5.40%	5.81%	0.08%	0.29%	5.14%

Table 2: Data underlying Figure 7

3.1.2. Analyzing 40-years saving plans

Figure 8 depicts the final capital of 391 (overlapping) 480-months *IDC*-saving plans with constant equity ratio $\beta = 0\%$, 50% and 100% - shown as solid lines. The corresponding final capitals for *CDC*-pension plans with $\theta_\Delta = 2\%$ and $\beta = 0\%$, 50% and 100% are shown as broken lines. The difference between the solid and the broken line indicates the degree of intergenerational transfer of assets.

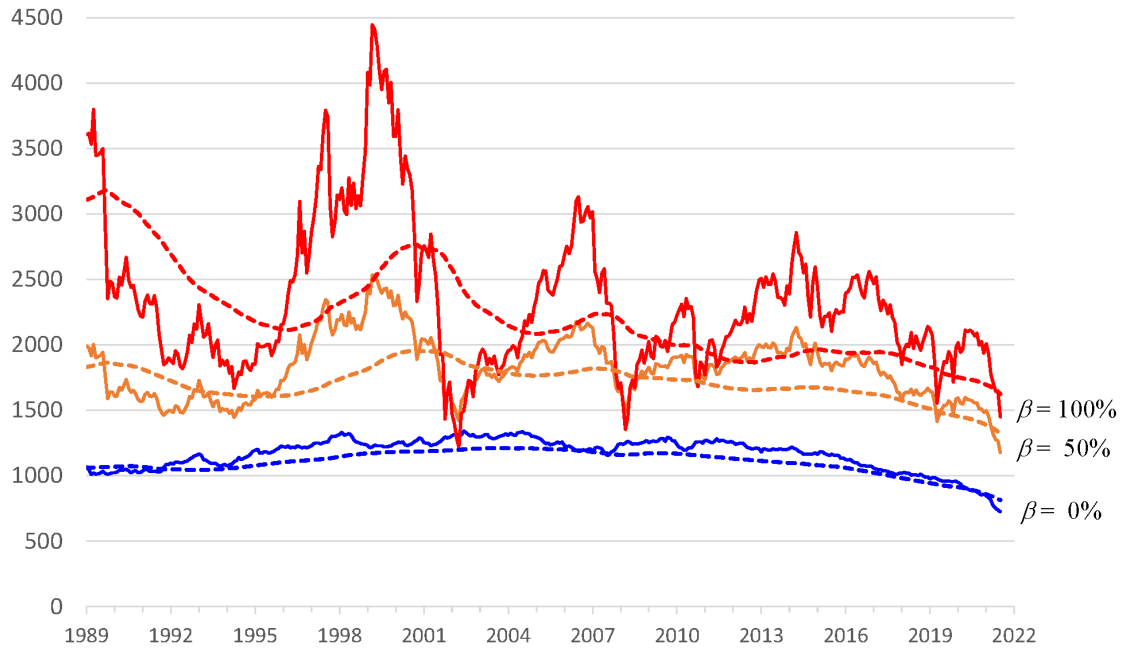
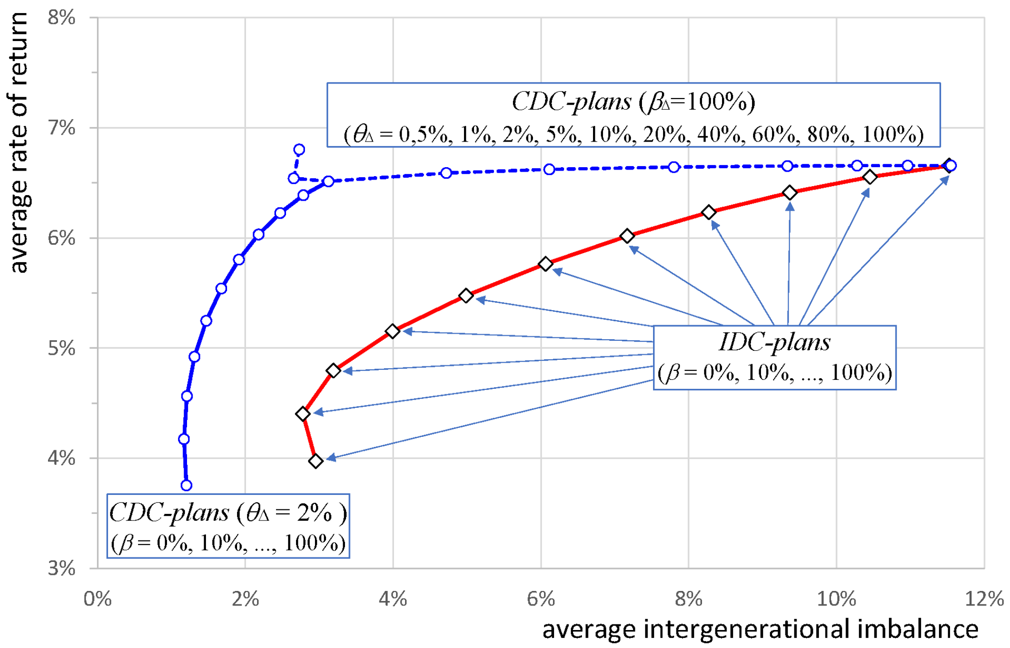


Figure 8. Final capital of 40-year saving plans for different *IDC*-/ *CDC*-variants; the *x*-axis shows the maturity date

The heavy ups and downs for *IDC*-saving plans with $\beta = 100\%$ indicate that even for a long-term saving plan it is nearly impossible to predict the outcome. As pointed out we want to evaluate long-term saving processes under the aspect of *intergenerational fairness*, having in mind, that a capital funded system should enable a fair participation in the economic capital. To this end we introduced the risk measure *intergenerational imbalance* (*IGI*) – in relation to the rate of return, – cf. the risk-return profiles in Figure 9. There is some resemblance to the risk-return profiles in Figure 6. Note that the range of rates of return for *IDC*-saving plans in Table 3 (from 3.97% to 6.66%) smaller than the range in Table 1 (from 3.07% to 7.07%). This is due to the fact that in Table 3 we analyze the rates of return of 480-months saving periods and that the rate of return of each of these is already a weighted average. The intergenerational risk transfer is particularly strong with respect to the *maximal IGI*. Here we observe that *increasing* β (from 0% to 60%) comes along with a *decreasing* of the *maximal IGI*. With respect to the *CDC*($\beta = 100\%$, θ_Δ) saving plans we notice for $\theta_\Delta = 0.5\%$ an abnormal risk-return profile. Again, this is just the consequence that the mean reverting effect is to low.



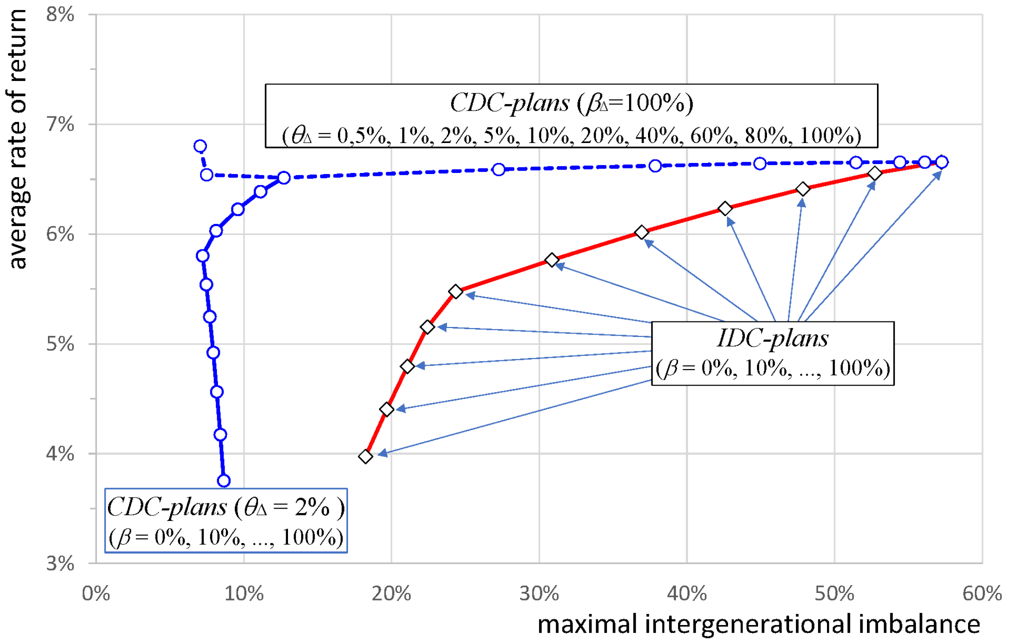


Figure 9. Average (top) and maximal (bottom) *intergenerational imbalance* (*IGI*) for *IDC*- and *CDC*-saving plans.

IDC-plan				*CDC* ($\theta_\Delta = 2\%$)				*CDC* ($\beta = 100\%$)			
β	rate of return	mean *ICI*	max *ICI*	β	rate of return	mean *ICI*	max *ICI*	θ_Δ	rate of return	mean *ICI*	max *ICI*
0%	3.97%	2.95%	18.24%	0%	3.75%	1.20%	8.64%	0.5%	6.80%	2.73%	7.03%
10%	4.40%	2.78%	19.67%	10%	4.17%	1.17%	8.41%	1%	6.54%	2.66%	7.47%
20%	4.79%	3.19%	21.07%	20%	4.56%	1.21%	8.17%	2%	6.51%	3.13%	12.70%
30%	5.15%	3.99%	22.43%	30%	4.92%	1.31%	7.93%	5%	6.59%	4.72%	27.26%
40%	5.48%	4.99%	24.34%	40%	5.25%	1.47%	7.69%	10%	6.62%	6.11%	37.85%
50%	5.76%	6.06%	30.86%	50%	5.54%	1.68%	7.45%	20%	6.64%	7.80%	44.95%
60%	6.02%	7.17%	36.94%	60%	5.80%	1.91%	7.20%	40%	6.65%	9.34%	51.44%
70%	6.23%	8.27%	42.59%	70%	6.03%	2.18%	8.11%	60%	6.66%	10.28%	54.41%
80%	6.41%	9.37%	47.85%	80%	6.23%	2.47%	9.59%	80%	6.66%	10.96%	56.10%
90%	6.55%	10.45%	52.72%	90%	6.39%	2.79%	11.13%	100%	6.66%	11.55%	57.25%
100%	6.66%	11.52%	57.24%	100%	6.51%	3.13%	12.70%				

Table 3: Data underlying Figure 9

The *ICI*-risk measure only depends on the final capital. In the following we evaluate the risk mitigating character of *CDC*-plans with respect to *volatility*, *maximum drawn* (*MDD*) and *maximum loss duration* (*MLD*) that evaluate the accumulation process. Note that these risk indicators are aggregates of 391 overlapping saving processes. The perspective of these risk indicators is that of the saver, who only looks at her or his saving plan – he or she makes no comparison to other generations savers.

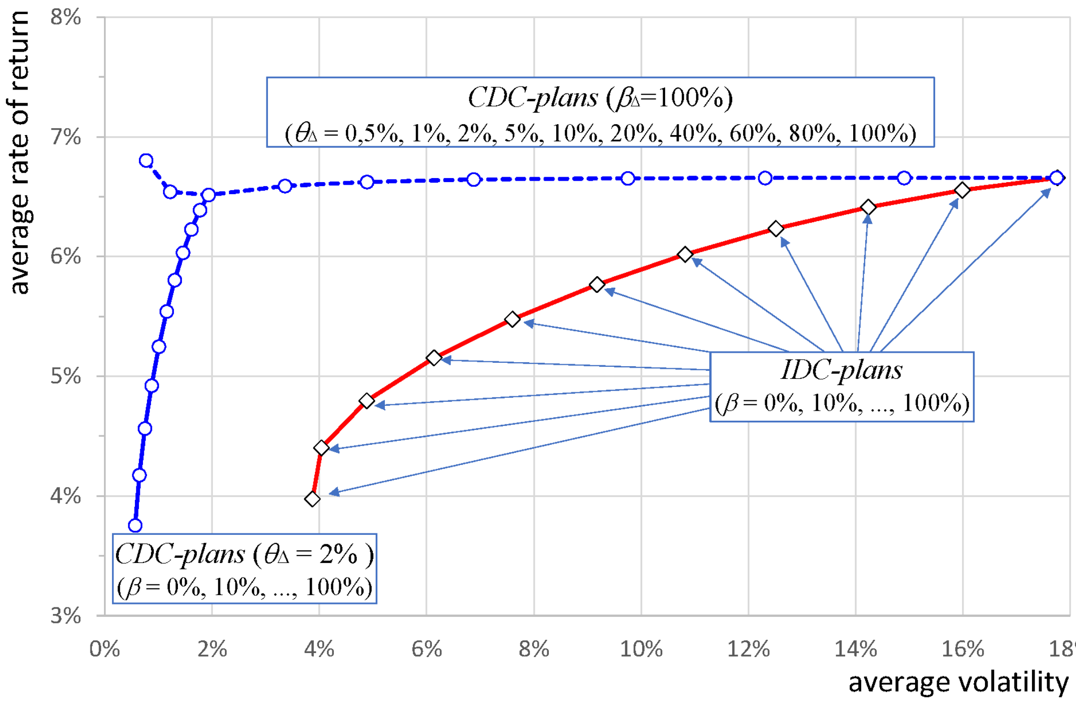


Figure 10. Average *rate of return* and average *volatility* for *IDC*- and *CDC*-saving plans.

Const-Mix-Portfolio		
β	average rate of return	average *Vola*
0%	3.97%	3.87%
10%	4.40%	4.04%
20%	4.79%	4.88%
30%	5.15%	6.14%
40%	5.48%	7.60%
50%	5.76%	9.18%
60%	6.02%	10.82%
70%	6.23%	12.51%
80%	6.41%	14.24%
90%	6.55%	15.99%
100%	6.66%	17.76%

CDC(θ_Δ = 2%, β)		
β	average rate of return	average *Vola*
0%	3.75%	0.57%
10%	4.17%	0.64%
20%	4.56%	0.75%
30%	4.92%	0.87%
40%	5.25%	1.01%
50%	5.54%	1.15%
60%	5.80%	1.30%
70%	6.03%	1.45%
80%	6.23%	1.61%
90%	6.39%	1.77%
100%	6.51%	1.93%

CDC(β = 100%, θ_Δ)		
θ_Δ	average rate of return	average *Vola*
0.5%	6.80%	0.77%
1%	6.54%	1.22%
2%	6.51%	1.93%
5%	6.59%	3.36%
10%	6.62%	4.89%
20%	6.64%	6.88%
40%	6.65%	9.75%
60%	6.66%	12.31%
80%	6.66%	14.90%
100%	6.66%	17.75%

Table 4: Data underlying Figure 10

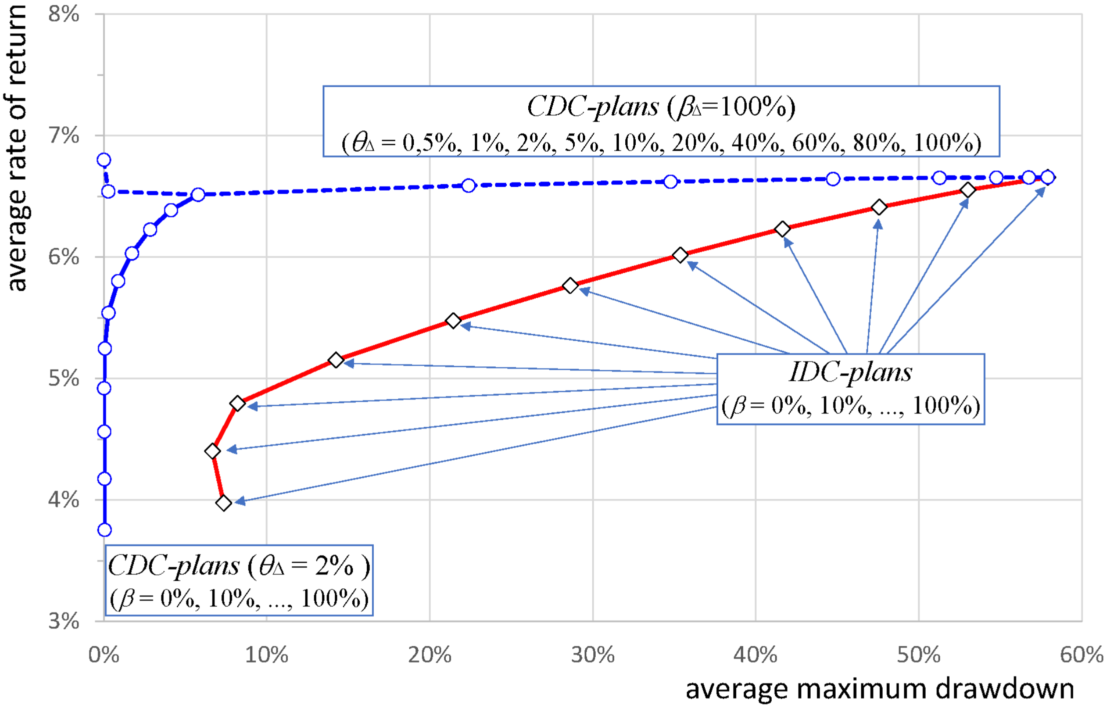


Figure 11. Average *rate of return* and average *MDD* for *IDC*- and *CDC*-saving plans.

Const-Mix-Portfolio		
β	average rate of return	average *MDD*
0%	3.97%	7.36%
10%	4.40%	6.67%
20%	4.79%	8.20%
30%	5.15%	14.25%
40%	5.48%	21.45%
50%	5.76%	28.62%
60%	6.02%	35.38%
70%	6.23%	41.63%
80%	6.41%	47.57%
90%	6.55%	53.01%
100%	6.66%	57.92%

$CDC(\theta_\Delta = 2\%, \beta)$		
β	average rate of return	average *MDD*
0%	3.75%	0.049%
10%	4.17%	0.035%
20%	4.56%	0.026%
30%	4.92%	0.017%
40%	5.25%	0.062%
50%	5.54%	0.29%
60%	5.80%	0.88%
70%	6.03%	1.73%
80%	6.23%	2.84%
90%	6.39%	4.13%
100%	6.51%	5.79%

$CDC(\beta = 100\%, \theta_\Delta)$		
θ_Δ	average rate of return	average *MDD*
0.5%	6.80%	0.003%
1%	6.54%	0.27%
2%	6.51%	5.79%
5%	6.59%	22.38%
10%	6.62%	34.77%
20%	6.64%	44.73%
40%	6.65%	51.26%
60%	6.66%	54.75%
80%	6.66%	56.74%
100%	6.66%	57.89%

Table 5: Data underlying Figure 11

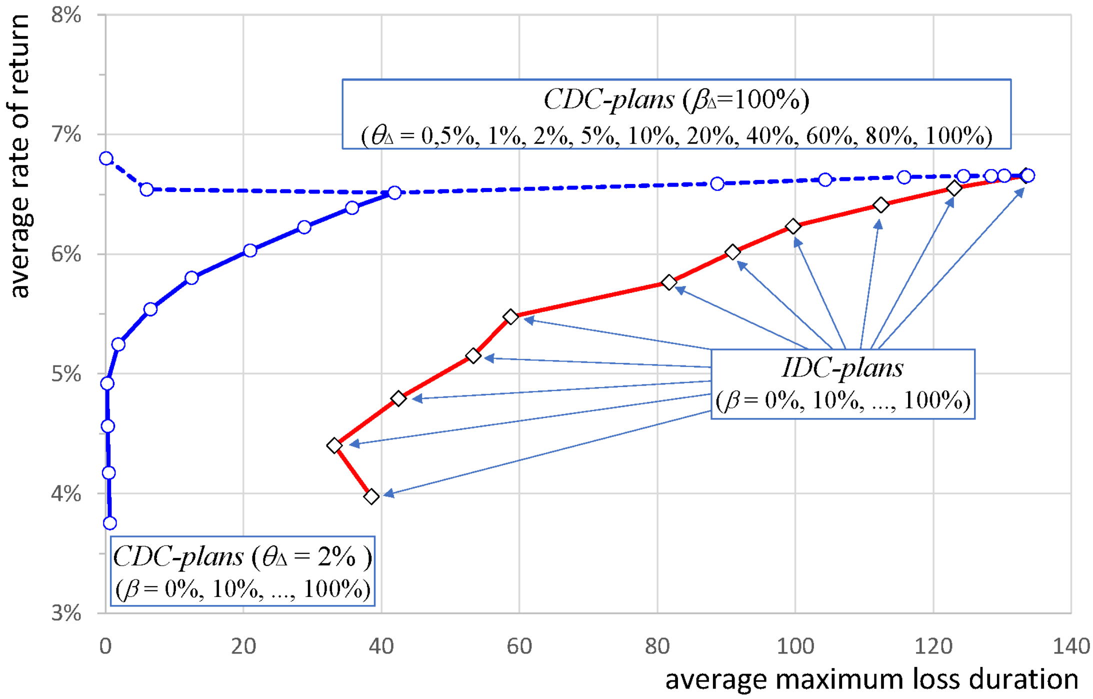


Figure 12. Average *rate of return* and average *MLD* for *IDC*- and *CDC*-saving plans.

Const-Mix-Portfolio		
β	average rate of return	average *MLD*
0%	3.97%	38.53
10%	4.40%	33.18
20%	4.79%	42.48
30%	5.15%	53.32
40%	5.48%	58.76
50%	5.76%	81.73
60%	6.02%	90.91
70%	6.23%	99.68
80%	6.41%	112.41
90%	6.55%	123.02
100%	6.66%	133.40

CDC(θ_Δ = 2%, β)		
β	average rate of return	average *MLD*
0%	3.75%	0.64
10%	4.17%	0.45
20%	4.56%	0.32
30%	4.92%	0.25
40%	5.25%	1.84
50%	5.54%	6.51
60%	5.80%	12.47
70%	6.03%	20.95
80%	6.23%	28.80
90%	6.39%	35.74
100%	6.51%	41.90

CDC(β = 100%, θ_Δ)		
θ_Δ	average rate of return	average *MLD*
0.5%	6.80%	0.08
1%	6.54%	5.94
2%	6.51%	41.90
5%	6.59%	88.67
10%	6.62%	104.30
20%	6.64%	115.76
40%	6.65%	124.37
60%	6.66%	128.39
80%	6.66%	130.28
100%	6.66%	133.71

Table 6: Data underlying Figure 12

4. Analyzing *CDC*-parameters

4.1.1. Intergenerational risk sharing parameter θ_Δ

We first consider the case $\alpha_\Delta = 0$. Then for *ERP* =5%, $\sigma_M = 20\%$ and $\beta(t) \equiv \beta_s$ we get $\mu_P^e(t) = \mu_s(t) + \Delta\,\beta_s(0.05 - 0.02\beta_s)$. Thus $\left(\mu_P^e(t)\right)$ and $\left(\mu_s(t)\right)$ have the same (annualized) volatility, namely $\sqrt{\Delta} \cdot StDev\left(\mu_s(t) : 0 \le t \le T\right) = 0.7270\%$.

Since $\eta(t) = \mu_P^e(t) + \theta_\Delta\,\hat{\rho}(t)$ we get $\eta(t) = \mu_P^e(t)$ for $\theta_\Delta = 0$, thus volatility of $\eta(t)$ and $\mu_s(t)$ are identical. For $\theta_\Delta = 1$ we have $\eta(t) = \mu_P^e(t) + \hat{\rho}(t)$ and by **Eq. 3** we get $\eta(t) = \mu_P(t) + \mu_s(t) - \mu_s(t - \Delta)$. Thus, the declaration for $[t, t+\Delta]$ coincides with the observed portfolio performance for $[t-\Delta, t]$ modified by the observed change of the government bond yield. Since the term $\mu_s(t) - \mu_s(t-\Delta)$ is small we conclude that for $\theta_\Delta = 1$ the volatility of $\eta(t)$ is very close to the volatility of the DAX_P-performance.[14] Figure 13 depicts the relation between θ_Δ and the annualized volatility of $(\eta(t))$ for three levels von β.

[14] For β=100%, θ_Δ=1: Vola($\eta(t)$) = 17.7657%; Vola($\mu_P(t)$) = 17.8304%.

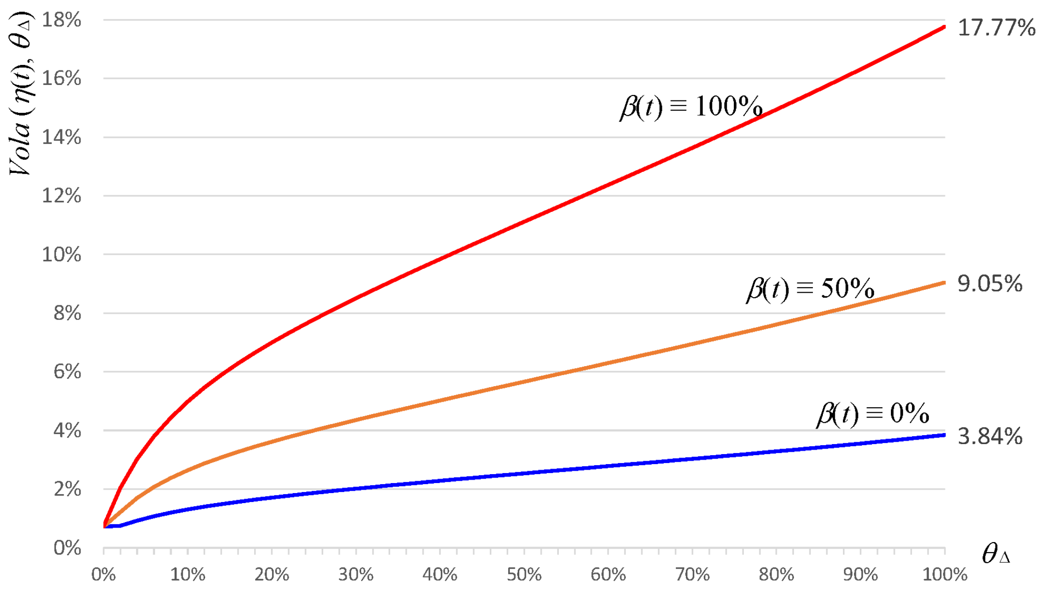


Figure 13: Annualized volatility of the $\eta(t)$-paths as a function of $\theta_\Delta \geq 0$ for different equity ratios.

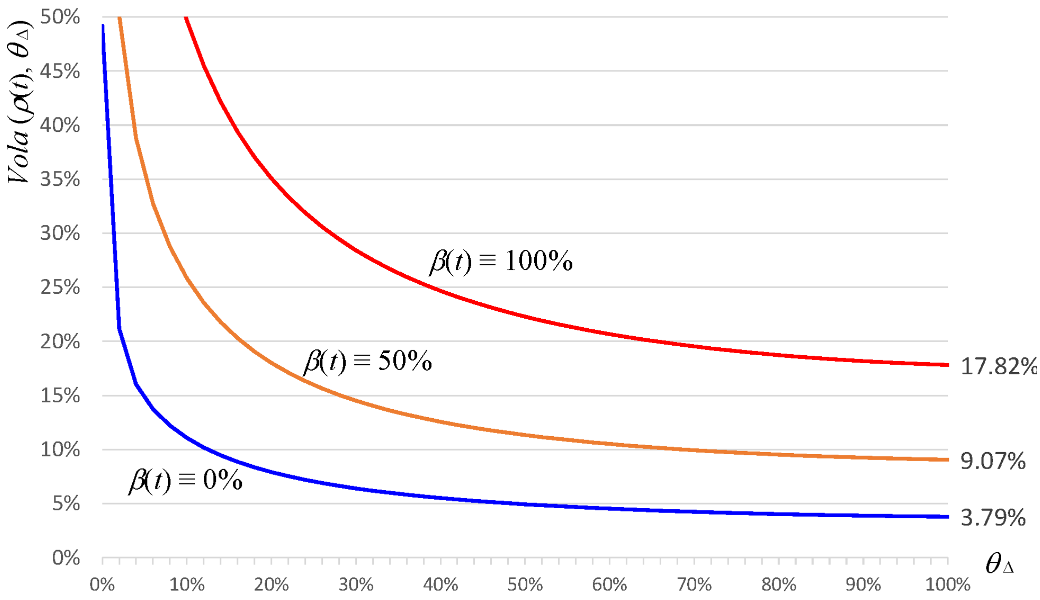


Figure 14: Annualized volatility of the $\hat{\rho}(t)$ -paths as a function of $\theta_\Delta \geq 0$ for different equity ratios.

Comparing Figure 14 with Figure 13 we see that a reduction of the volatility of the $\eta(t)$-paths by lowering the risk sharing parameter θ_Δ is only possible if we accept a more volatile reserve ratio. Note that for $\theta_\Delta = 1$ we have $\eta(t) = \mu_P^e(t) + \hat{\rho}(t)$ and thus the volatility of $\left(\hat{\rho}(t)\right)$ is very close to the volatility of $\left(\eta(t)\right)$.

4.1.2. *Asset adjustment parameter α_Δ*

The following Proposition is the motivation for our *LM-Rule* $\beta(t) := \beta_s + \alpha_\Delta\left(\rho(t) - \rho_s\right)$ and it helps to calibrate the parameters β_s and α_Δ . Suppose at time t we want to control the "*ruin probability*" that the reserve $\rho(t+\Delta)$ falls behind a given minimum level $\rho_{\min}$, then the following holds:

Proposition [15]

If we assume that $\mu_P(t+\Delta) - \mu_P^e(t) \overset{distr}{=} \beta(t)\,\sigma_M\,(W_{t+\Delta} - W_t)$,[16] we get

$$\mathbb{P}\left(\hat{\rho}(t+\Delta) \le \rho_{\min} - \rho_s \middle| \hat{\rho}(t)\right) = \Phi\left(-\frac{\rho_s - \rho_{\min} + (1-\theta_\Delta)\,\hat{\rho}(t)}{\sqrt{\Delta}\;\beta(t)\,\sigma_M}\right) \text{ and}$$

$$\mathbb{P}\left(\rho(t+\Delta) \le \rho_{\min} \middle| \rho(t)\right) \le \varepsilon \iff \beta(t) \le \beta_s + \alpha_\Delta\left(\rho(t) - \rho_s\right), \qquad \textbf{(Eq 4)}$$

with $\beta_s := \dfrac{\rho_s - \rho_{\min}}{\sqrt{\Delta}\;\sigma_M\,u_{1-\varepsilon}}$, $\alpha_\Delta := \dfrac{1-\theta_\Delta}{\sqrt{\Delta}\;\sigma_M\,u_{1-\varepsilon}}$ and $u_{1-\varepsilon} := \Phi^{-1}(1-\varepsilon)$, the 1-$\varepsilon$ quantile of the standard normal distribution. Here (W_t) denotes a (discrete) standard Wiener process.

◊

Remark: The assumption $\mu_P(t+\Delta) - \mu_P^e(t) \overset{distr}{=} \beta(t)\,\sigma_M\left(W_{t+\Delta} - W_t\right)$ is daring for several reasons. Firstly, even for $\beta(t) = 0$ $\mu_P(t+\Delta) - \mu_P^e(t)$ bears an interest rate risk, secondly, the monthly

[15] The proof is straight forward and left to the reader.

[16] "$\overset{distr}{=}$" stands for „has the same distribution as"

returns of a pure equity portfolio are only approximately normal-distributed. Thirdly, given $\rho(t)$ the distribution of $\mu_P(t+\Delta)-\mu_P^e(t)$ is *not* independent of the *t*-history – in other words, real world capital markets are not *efficient*.[17]

Example: For $\sigma_M = 20\%$, $\rho_s - \rho_{\min} = 20\%$, $\varepsilon = 1\%$ and $\theta_\Delta = 2\%$ we get

$$u_{1-\varepsilon} = 2.3263\ ,\ \beta_s = \frac{\rho_s - \rho_{\min}}{\sqrt{\Delta}\ \sigma_M\ u_{1-\varepsilon}} = 148.91\%\ ,\ \alpha_\Delta = \frac{1-\theta_\Delta}{\sqrt{\Delta}\ \sigma_M\ u_{1-\varepsilon}} = 7.2965\ .$$

This means that – at least in theory - even a debt financed equity investment ($\beta > 100\%$) would be allowed, if we require that in only 1 of 100 months the reserve gap falls below - 20%.

As can be seen from Figure 15 the "*ruin probabilities*" (here: the proportion of case where the reserve gap falls below – 20%) are much higher than the Proposition above suggests. The reason is that log-returns of equity markets are not normally distributed.[18]

[17] For example, "irrational exuberances" - analyzed by Nobel laureate Robert Shiller (Shiller 2014) - strongly indicate that equity markets are not efficient.
[18] If we take $\sigma_M = 40\%$ instead of $\sigma_M = 20\%$ in the example above the ruin probabilities according ***Eq* 4** approximately correspond to the backtesting results in Figure 15.

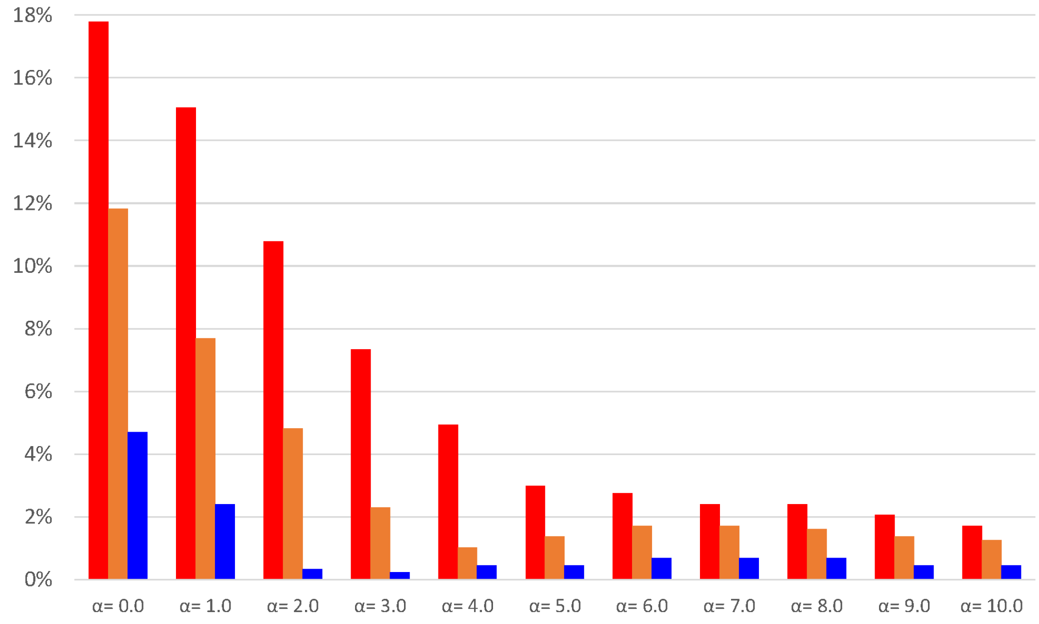


Figure 15: Probability that $\hat{\rho}(t)$ falls below -20% for different α_Δ-levels with underlying *ALM* parameters θ_Δ = 2% and β_s = 100%/ 75%/ 50% (red/ orange/ blue column).

Backtesting results in Figure 15 show that the *LM*-parameter α_Δ is a decisive instrument to manage the reserve gap. Figure 15 also indicates that high α_Δ-levels could be counterproductive at least for β_s = 75% or β_s = 50%.

5. Conclusion and outlook

Our analysis shows that *collective defined contribution* (*CDC*) schemes are more than just a nice idea. Based on the theoretical model presented in (Goecke, 2013) we could prove that the risk return profile for *CDC* plans is much better than that for *DC* plan. Even in a worst case scenario *CDC* plans perform better than individual *DC* plans. Clearly, there is no guarantee that an excellent performance observed in the past will recur in future. But it should be stressed that our analysis is based on observations of $72^1/_2$ years. Within this time span, we have observed extreme situations such as the oil price crisis 1973, stock market crashes and bubbles, high inflation rates, and extreme short and long term interest rates.[19] The strength of the *CDC* plan is that it is self adjusting due to resilience factors in the *ALM* rules. With respect to the fundamental objective to ensure a fair participation in production factor capital, a *CDC* plan is superior to a pension plan with an interest rate guarantee. Looking back into the economic history, we should realize that *long term* interest rate guarantees either are worthless or are unbearable for the warrantor. Pension systems must be adjustable to the economic reality!

It has been criticized that a *CDC* plan is just a zero-sum game: the advantage for one generation of savers is the disadvantage for the other. This view totally ignores the principle of risk sharing and insurance. A *CDC* scheme is an "insurance" contract where the saver pays a premium (in form of contributions into the collective reserve) and receives benefits (in form of payments out of the collective reserve). In *A Theory of Justice* John Rawls introduced the concept of *fairness* under the *veil of ignorance*.[20] Following Rawls' theory a transgenerational contract between savers will be regarded as *fair*, if the savers were to agree upon this contract provided they did not know in advance which generation they belong to – i.e. *under the veil of ignorance*. From this perspective a *CDC* plan is fair. However, it is also clear that a saver might feel unfairly treated if she or he was obliged to enter a heavily underfunded *CDC* scheme.

CDC schemes are not an all-purpose answer to the pension challenge in an aging society. There are apparent limitations. For example a *CDC* plan can only work if the participants cannot withdraw money at will. Contractual compliance is essential for intergenerational risk transfer. That does not mean that *CDC* plans are Ponzi plans. Whenever the flow of new

[19] The 1-month money market rates ranged between 13.33% (Sept. 1973) and – 0.6% (Dec. 2021).
[20] Cf. (Rawls 1991), in particular Section 24 (pp. 118ff.) and Section 44 (pp. 251ff.)

entrants stops, a *CDC* plan can be converted into a simple *DC* plan by setting the target reserve ratio to zero.

Our backtesting analysis considered only the accumulation phase. We assumed that the wealth at retirement is paid out and reinvested outside the fund. An apparent extension of the model is to include a decumulation phase and then analyze the pension risk, e.g. the volatility of pensions in payment. It would be equally interesting to analyze population dynamics (growing or shrinking generations, winding off) in a *CDC* scheme.

We hope at least, that our analysis has added one more argument in favour of *CDC* plans as a good alternative to *DC* and *DB* plans.

Appendix: Data Description and Calibration of *ERP* and σ_M

We take *REXP* as a proxy for a portfolio of German government bonds and *DAX* as a proxy for a well-diversified portfolio of German equities. *REXP* and *DAX* are both performance indices calibrated such that *REXP*(end of year 1987) = 100 and *DAX*(end of year 1987) = 1000.

The index *REXP* is calculated on the basis of a portfolio 30 different fictious German government bonds with coupon rates 6%, 7.5% and 9% and maturities between 1 and 10 years.[21] The average *Macauly duration* is 5.02, 4,81 and 4.62 for interest rates 0%, 3% and 6%, respectively. Clearly, an investment into a *REXP*-Portfolio (or in German government bonds) is not risk-free in the strict sense since from month to month a pension asset manager will experience unexpected gains or losses from volatile market interest rates leveraged by the duration of the portfolio. Consistent to our pragmatic definition of a risk-free asset, we will substitute the constant risk-free interest rate of the *c.t.* model by the current yield of outstanding government bonds (described in detail below), which we interpret as the expected return of a *REXP*-investment.

DAX is a weighted performance index comprising the 40 biggest German joint stock companies.[22]

End of month index values for *REXP* and *DAX* are provided by Deutsche Bundesbank from end of Jan. 1967 (*REXP*) and end of Dec. 1987 (*DAX*) onwards.[23] The end of month index values (based on the last fixing on the last trading date of the particular month) is considered

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[21] Deutsche Bundesbank (2021); Statistische Fachreihe Kapitalmarktkennzahlen, Januar 2022, p. 17f.
[22] Cf. Deutsche Börse AG (2022).
[23] Time series BBK01.WU3141 and BBK01.WU046A

to be identical with the index value of the first trading date of the following month just before the first fixing.

Remark on notation: We use the time index t representing any point in time between $t_0 = 0$ (representing the beginning of the first trading day of Jan. 1950) and $T = 870\Delta$ (representing the beginning of the first trading day of July 2022). Since we have a discrete time (*d.t.*) model based on monthly data, we sometimes write $t = 01.mm.yyyy$ [24] instead of $t = k\Delta$ [25] just to make transparent what date t is referring to.

For *REXP*- and *DAX*-index values not published by Deutsche Bundesbank, we use backward projections from different sources:

Backward Projection for $DAX(t)$ ($t_0 \leq t \leq 01.01.1988$)

We use the backward projection of Gielen (1994),[26] adjusted such that the index value matches $DAX(01.01.1988) = 1000$.

Backward Projection for $REXP(t)$ ($t_0 \leq t \leq 01.01.1967$)

For $01.01.1950 \leq t \leq 01.01.1967$ we use the following formula for backward projection

$$REXP(t) = REXP(t+\Delta)\cdot\left(1+i_s(t)\right)^{\frac{-1}{12}}\cdot\left(\frac{1+i_s(t+\Delta)}{1+i_s(t)}\right)^{D}, \qquad \textbf{(Eq A1)}$$

with $REXP(01.02.1967) = 21.24$,[27] $i_s(t) = \textit{risk free interest rate}$ observed at time t and $D = 4.80$.[28]

We interpret $i_s(t)$ to be the yield of an investment into German government bonds held to maturity invested at time t. However, the actual return on investment, calculated at time

[24] The first of a month is identified with the beginning of the first trading date of this month.
[25] k counts the number of months after 01.01.1950, i.e. $k = 12\cdot(yyyy\text{-}1950) + mm\text{-}1$
[26] Gielen, Gregor (1994): Können Aktienkurse noch steigen? Langfristige Trendanalyse des deutschen Aktienmarktes, Gabler-Verlag, Wiesbaden 1994
[27] *REXP* index value for end of Jan. 1967 according to Deutsche Bundesbank data.
[28] 4.8 equals Macauly-Duration of *REXP*-portfolio for an interest rate of 3.19%.

$t+\Delta$, depends also on the interest rate $i_s(t+\Delta)$ and the duration of the bond portfolio. We stipulate that the market value at time t+1 of a German government bond investment of 100€ at time t is $100€\cdot\left(1+i_s(t)\right)^{1/12}\cdot\left(\frac{1+i_s(t)}{1+i_s(t+\Delta)}\right)^D$. This is the motivation for (Eq A1).

Risk free Interest Rates: $i_s(t)$ ($t_0 \le t \le T$):

- 01/1950 ≤ t ≤ 01/1953: Mean value of Central Bank lombard and discount rate[29]
- 02/1953 ≤ t ≤ 01/1954: Current yield of outstanding public bonds (5.0% coupon)[30]
- 02/1954 ≤ t ≤ 02/1956: Current yield of outstanding public bonds (5.5% coupon)[31]
- 03/1956 ≤ t ≤ 03/1960: Current yield of outstanding public covered bonds[32]
- 04/1960 ≤ t ≤ 07/2022: Current yield of outstanding German government bonds.[33]

Note that e.g. $i_s(t)$ is the average of observed yields of the *foregoing* month; $i_s(t)$ can be observed time t.

Money-Market Index $MMI(t)$

We calculate a *Money Market* index based on the following money market rates $i_{MM}(t)$ published by Deutsche Bundesbank:

- 01/1950 ≤ t ≤ 01/1960: day-to-day money (average of minimum and maximum)[34]
- 02/1960 ≤ t ≤ 01/1999: 1-month-money (monthly average)[35]
- 02/1999 ≤ t ≤ T: 1-month-EURIBOR (monthly average).[36]

We set $MMI(t = 01/1950)$:= 100 and define recursively

$$MMI(t+\Delta) = MMI(t)\cdot\left(1+i_{MM}(t)\right)^{\frac{1}{12}}.$$

[29] Deutsche Bundesbank, Time Series BBK01.SU112 and BBK01.SU113
[30] Calculated from published market values for covered bonds; Wirtschaft und Statistik, Montsberichte 06/1953 (p. 256*), 12/1953 (p. 676*) and 12/1954 (p. 652*) https://www.statistischebibliothek.de/mir/receive/DESerie_mods_00000012.
[31] Evaluation of Deutsche Bundesbank Monthly Reports April – Dec. 1956
[32] Deutsche Bundesbank, Time Series BBSIS.M.I.UMR.RD.EUR.MFISX.B.A150.A.R.A.A._Z._Z.A
[33] Deutsche Bundesbank, Time Series BBSIS.M.I.UMR.RD.EUR.S1311.B.A604.A.R.A.A._Z._Z.A
[34] Deutsche Bundesbank, Time Series BBK01.SU0102 and BBK01.SU0103
[35] Deutsche Bundesbank, Time Series BBK01.SU0104
[36] Deutsche Bundesbank, Time Series BBK01.SU0310

Implicitly we hereby assume that at time t we can invest in a 1-month money market paper with interest rate $i_{MM}(t)$.

Consumer Price Index $CPI(t)$ for t = 01.01.1949 to t = 01.07.2022:

$CPI(t)$ is the concatenation of the following two time series:

- 01.01.1949 ≤ t ≤ 01.01.1991: Consumer Price Index for West Germany [37]
- 01.02.1991 ≤ t ≤ 01.07.2022: Consumer Price Index[38].

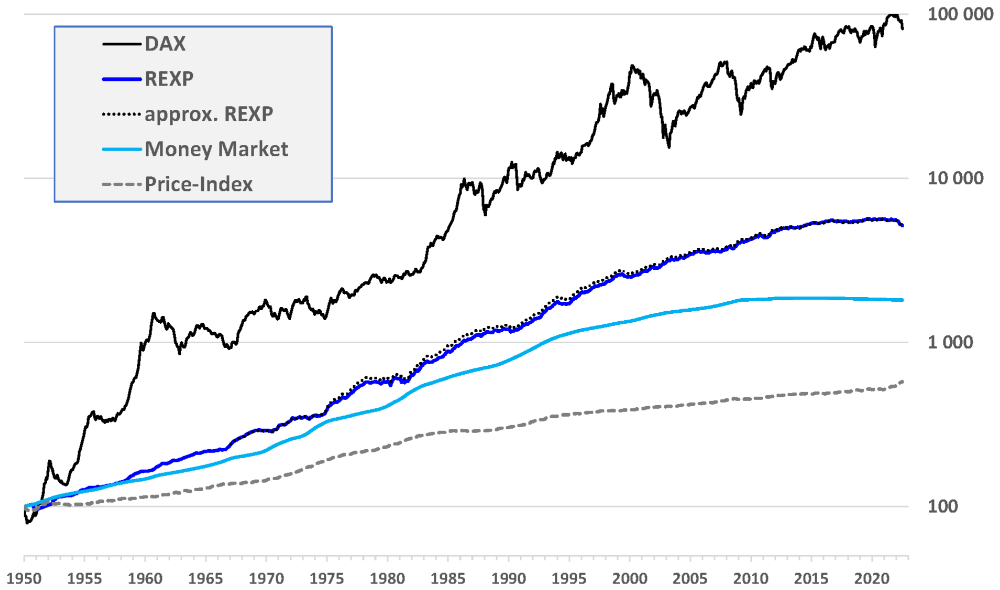

Figure A1: *Price Index*, Performance of a *DAX*-, *REXP*- and a *Money Market*-investment, normalized at 100 for t_0 =1.1.1950, log-scaled

Figure A1 illustrates the performance of a *DAX-, REXP*- and *MMI*-investment starting with an initial capital of 100 on 01.01.1950. To illustrate the reverse projection for *REXP* in Figure A1 we have added a *forward* projection of *REXP* (from 01/1967 onwards) using formula (Eq

[37] Statistisches Bundesamt (2021), Preise, Verbraucherpreise für Deutschland, Lange Reihen ab 1948, „Früheres Bundesgebiet, Preisindex für die Lebenshaltung, 4-Personen-Haushalte von Arbeitern und Angestellten mit mittlerem Einkommen, index basis = 100 (average for year 1995). This time series has been rescaled such that the index value for 01/1991 equals 86.9; the rescaled index value are rounded to two decimal points.
[38] Statistisches Bundesamt, Fachserie 17, Reihe 7 (time series 61111-0002), index basis = 100 (average for year 2015)

A1) – see dotted line. Obviously *REXP* can well be approximated by using only one interest rate (namely $i_s(t)$ as described above) and a suitable estimation of the average (Macauly-) duration.

<u>Price adjusted indices $REXP_p(t)$, $DAX_p(t)$ and $MMI_p(t)$ for t = 01.01.1950 to 01.07.2022:</u>

$$REXP_p(t) = REXP(t)\,\frac{CPI(t_0)}{CPI(t)},\ DAX_p(t) = DAX(t)\,\frac{CPI(t_0)}{CPI(t)},\ MMI_p(t) = MMI(t)\,\frac{CPI(t_0)}{CPI(t)}$$

with t_0 = 01.01.1950 $\leq t \leq$ 01.07.2022

<u>Price adjusted interest rates $\mu_s(t)$ and $\mu_s(t)$ for t = 01.01.1950 to 01.07.2022:</u>

$$\mu_s(t) = \frac{1}{12}\ln\left((1+i_s(t))\,\frac{CPI(t-12\Delta)}{CPI(t)}\right)$$

We use the log-interest rate on monthly basis to simplify the notation. We interpret $\mu_s(t)$ as the *expected* real return of a risk-free investment at time t for the following month $[t, t+\Delta]$ on the basis of the observed yield $i_s(t)$ and the experienced depreciation of the foregoing year.[39] $\mu_s(t)$ can be calculated on the basis of information up to time t.

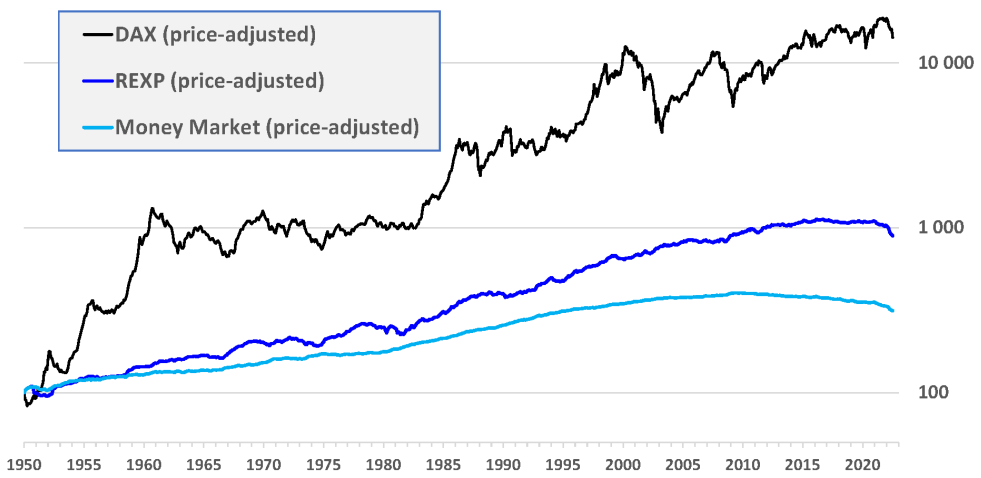

Figure A2: Performance of a price-adjusted *DAX*-, *REXP*- and *Money Market*-investment of 100 (log-scaled).

[39] We prefer a price adjustment on a yearly basis to eliminate seasonal effect of the price index.

Figure A2 makes obvious, that in the last years a risk-free investment into *REXP* or *MMI* could not compensate inflation.

Calibration of σ_M and *ERP*

We calibrate σ_M and *ERP* as follows: $\boldsymbol{\sigma_M}$ **= 20%** and ***ERP* = 5%**, being constant for the backtesting period. In the following we want to show, that this calibration is at least *plausible*.

We need a calibration of σ_M and *ERP* to determine the expected return of a portfolio with equity ratio of $\beta(t)$, namely $\mu_P^e(t) = \mu_s(t) + \Delta\left(\beta(t)\,ERP - \tfrac{1}{2}\beta^2(t)\sigma_M^2\right)$ - cf. *LM-Rule* above.

The underlying stochastic model for a 100% equity ratio is

$$\mu_P(t) \overset{distr}{=} \mu_P^e(t) + \sigma_M\, Z_t^\Delta = \mu_s(t) + \Delta(ERP - \tfrac{1}{2}\sigma_M^2) + \sigma_M\, Z_t^\Delta\ , \qquad \textbf{(Eq A2)}$$

where (Z_t^Δ) are independent normally distributed random variables with $\mathbb{E}(Z_t^\Delta) = 0$ and $Var(Z_t^\Delta) = \Delta = \frac{1}{12}$. As our backtesting is based on *price adjusted* capital market data, σ_M and *ERP* have to be calibrated accordingly. Note that in (Eq A2) the *ERP* refers to the (expected) extra return on equities (DAX_p) over an investment into government bonds. Thus a plausible value for σ_M would be:

$$\sigma_M = \sqrt{12} \cdot StdDev\left(\ln\left(\frac{DAX_p(t)}{DAX_p(t-\Delta)} \right) - \mu_s(t) : \Delta \le t \le 870\Delta \right) = 17.8305\%\ .$$

The average excess (log-) return is $Mean\left(\ln\left(\frac{DAX_p(t)}{DAX_p(t-\Delta)} \right) - \mu_s(t) : \Delta \le t \le 870\Delta \right) = 0.3295\%$.

This, together with $\sigma_M = 17{,}8305\%$ gives the following estimation for *ERP*:

$$ERP = 12 \cdot 0.3295\% + \tfrac{1}{2}(17.8305\%)^2 = 5.5436\%\ .$$

If we calibrate σ_M and *ERP* on the basis of the last 480/ 360/ 240/ 120/ 60 months, we get

σ_M = 20.49% / 20.76% / 20.49% / 16.71% / 18.56% and

ERP = 6.62%/ 5.74% / 5.49% / 7.65% / 1.80%, respectively.

These numbers illustrate that there is no statistically stable calibration. Our choice of *ERP* = 5% is also motivated by "*The Rate of Return on Everything, 1870-2015*"[40]. The authors calculate for Germany (1950-2015) an excess real return on equities over bonds of 7.52% - 3.69% = 3.83%.[41]

[40] Cf. Jorda e.a. (2019)
[41] Cf. Jorda e.a. (2019), Table 4 and 5. The caculations of Jorda e.a. are based on averaged of annual returns and not – as we do – on averages of monthly log-returns.

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